Danbury Park Community Primary School

Mathematical Calculations Policy

The Teaching and Learning of Calculation Methods and Strategies at Danbury Park Community Primary School

Be wise, be happy, belong

Danbury Park Community Primary School
Written Calculations Policy

Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave primary school they:

- have a secure knowledge of number facts and a good understanding of the four calculations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Children should know and understand a compact standard method for each numerical calculation by the end of year 6. Each stage has a progression of difficulty starting with numbers that do not bridge ten (e.g. 27 + 12 rather than 27 + 14). There are lots of practical activities prior to formal written calculations.

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED, THEY ARE NOT REPLACED BY WRITTEN METHODS!

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four calculations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the calculations relate to one another and how the rules and laws of arithmetic are to be used and applied, continuing to use visual cues and practical apparatus to affirm understanding. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

Estimation

To aid their written methods all children will be required to estimate for all four calculations. Estimation provides a system for checking an answer (i.e. is the calculated answer sensible?). An estimate should be a mental calculation and should not require any formal working.

87 + 43 =
Est. 90 + 40 = 130

238 – 127 =
Est. 200 – 100 = 100

63 X 28
Est. 60 X 30 = 1800

587 X 62
Est. 600 X 60 = 36000

291 ÷ 3
Est. 270 ÷ 3 = 90 OR 300 ÷ 3 = 100

Addition

Stage 1: The empty number line

The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.

The empty number line helps to record the steps on the way to calculating the total.
Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

Key Stage 1

Children start using a number line with simpler sums that involve adding on in steps of one.

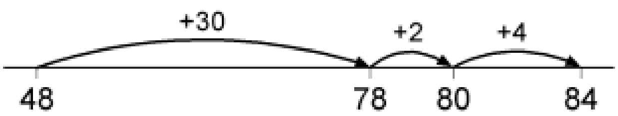
9 + 5 = 14

They will then progress onto using the number line in a more versatile way:

8 + 7 = 15

Key Stage 2

48 + 36 = 84



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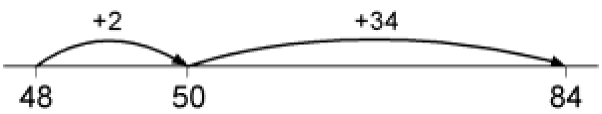


Stage 2: Partitioning

The next stage is to record mental methods using partitioning: add the tens and then the ones to form partial sums and then add these partial sums.
Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.
Children begin to solve problems that do not bridge the tens. For example:

36 + 33 =

T = 30 + 30 = 60

U = 6 + 3 = 9

60 + 9 = 69

Children will then progress onto more challenging additions that bridge the tens.

47 + 76 =

47 + 70 = 117

117 + 6 = 123

Stage 3: Expanded method in columns

Children then move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums the ones are added first.
The addition of the tens in the calculation 47 + 76 is described in the words ‘forty plus seventy equals one hundred and ten’.
The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Adding the units first:

```
   47
 + 76
 ----
   13   (7 + 6)
  110   (40 + 70)
 ----
  123
```

Stage 4: Column method

In this method, recording is reduced further. Carry digits are recorded below the line, using the words ‘carry ten’ or ‘carry one hundred’, not ‘carry one’.

Later, this is extended to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.

47	258	366
+ 76	+ 87	+458
123	345	824
11	11	11

Column addition remains efficient when used with larger whole numbers and decimals.
Once learned, the method is quick and reliable.

Subtraction

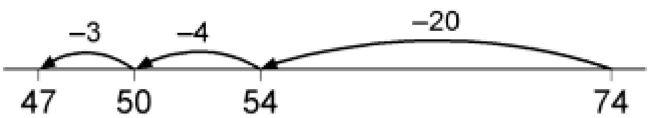
Stage 1: Using the empty number line

The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.
Steps in subtraction can be recorded on a number line. These often bridge through a multiple of 10.

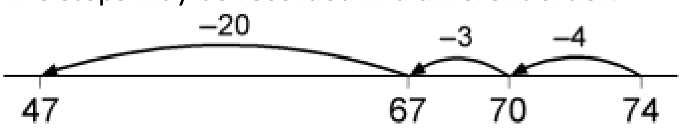
Key Stage One

15 - 7 = 8

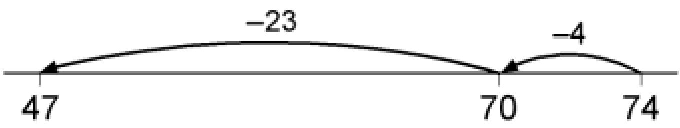
74 - 27 = 47 worked by counting back:



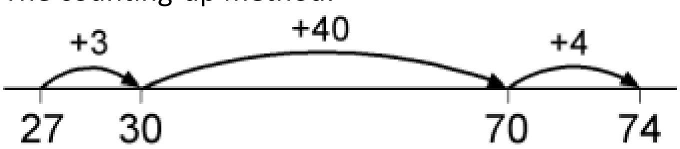
The steps may be recorded in a different order:



or combined:



The counting-up method:



Stage 2: Partitioning

Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.

Children begin by completing subtraction calculations that do not bridge the tens.
Subtraction can be recorded using partitioning:

74 - 27 =

74 - 20 = 54

54 - 7 = 47

Stage 3: Short column subtraction (decomposition)

The shorter more compacted method involves less working. Children are encouraged to **exchange** (avoid 'borrow') from the next column of place value when a subtraction cannot give a positive answer.

948 - 263 =

```
 8 1
 948
-263
----
 685
----
```

Multiplication

Multiplying by 10 or 100

Key understanding is that:

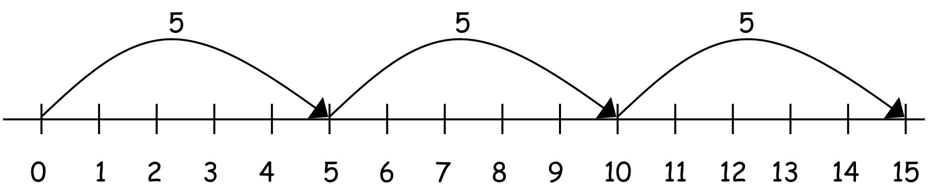
- the effect of multiplying by 10 is a shift in the digits one place to the left;
- the effect of multiplying by 100 is a shift in the digits two places to the left.

Avoid the use of the phrase, “add a zero.” Use: “digits move to the left and the places are filled using a zero.”

Stage 1: Repeated addition

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3.
Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5



Stage 2: Mental multiplication using partitioning

Mental methods for multiplying a two-digit number by a one-digit number can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.

Mental multiplication can also be recorded using partitioning:

$$\begin{aligned}14 \times 3 &= (10 + 4) \times 3 \\ &= (10 \times 3) + (4 \times 3) = 30 + 12 = 42\end{aligned}$$

$$\begin{aligned}43 \times 6 &= (40 + 3) \times 6 \\ &= (40 \times 6) + (3 \times 6) = 240 + 18 = 258\end{aligned}$$

Stage 2: The grid method

The grid method is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

×	7
30	210
8	56
	266

Stage 3: Expanded short multiplication

The next step is to represent the method of recording in a column format, but showing the working, drawing attention to the links with the grid method above.
Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.

```
  38
X  7
  56   (7 X 8)
 210   (7 X 30)
 266
```

Stage 4: Short multiplication

The recording is reduced further, with carry digits recorded below the line.
If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.

```
  38
×  7
 266
  5
```

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

Stage 5: Two-digit by two-digit products

The recording is reduced, showing the links to the grid method above. For this method the children will multiply the less significant digit first (i.e. multiply the ones, then the tens etc.)

	56	
x	27	
	42	7 x 6 = 42
	350	7 x 50 = 350
	120	20 x 6 = 120
	1000	20 x 50 = 1000
	1512	
	1	

The recording is further reduced. The carry digits in the partial products of 56 × 20 = 1120 and 56 × 7 = 392 are usually carried mentally.

	56	
x	27	
	392	56 x 7
	1120	56 x 20
	1512	
	1	

Stage 6: Three-digit by two-digit products

Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.

	3 2 7	
x	5 3	
	9 8_2 1	← 327 x 3
	1 6_1 3_3 5 0	← 327 x 50
	1 7 3 3 1	
	1 1	

Division

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.
e.g. If I know 3 x 7 = 21, what else do I know?
30 x 7 = 210, 300 x 7 = 2100, 3000 x 7 = 21 000, 0.3 x 7 = 2.1 etc

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Dividing by 10 or 100

Key understanding is that:

- the effect of dividing by 10 is a shift in the digits one place to the right;
- the effect of dividing by 100 is a shift in the digits two places to the right.

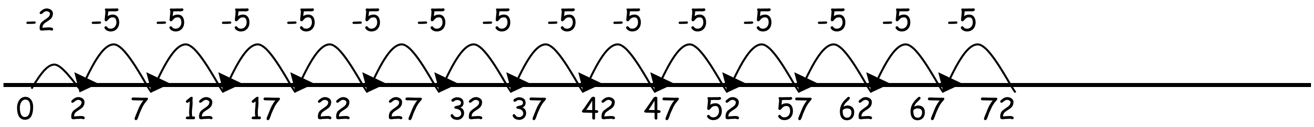
Avoid the use of the phrase, “take off a zero.” Use: “digits move to the right and the decimal point **does not move**.”

Stage 1: Repeated subtraction

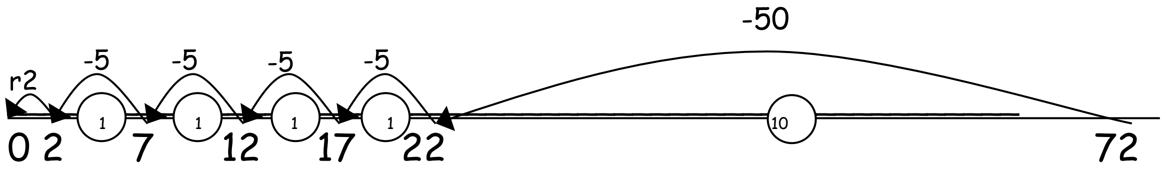
Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

Repeated subtraction

72 ÷ 5 =



Moving onto:



Stage 2: 'Expanded' method for TU ÷ U moving onto HTU ÷ by U

This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.
For two-digit numbers divided by a one-digit number there is a link to the mental method.
As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?'
Once the children understand and can apply the method, they should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.

This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.

Chunking is useful for reminding children of the link between division and repeated subtraction. However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.

97 ÷ 9 =

$$\begin{array}{rl} 9\overline{)97} & \\ -\underline{90} & 9\times10 \\ 7 & \end{array}$$

Answer: 10 r 7

196 ÷ 6 =

$$\begin{array}{rl} 6\overline{)196} & \\ -\underline{60} & \times10 \\ 136 & \\ -\underline{60} & \times10 \\ 76 & \\ -\underline{60} & \times10 \\ 16 & \\ -\underline{12} & \times2 \\ 4 & \end{array}$$

Answer: 32 r 4

Stage 3: Short division of HTU ÷ U

Short division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction.
Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

$$\begin{array}{r} 0\ \ 9\ \ 7 \\ 3\overline{)2\ \ 9\ {}^{2}1} \end{array}$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3.
The 97 written above the line represents the answer: 90 + 7, or 9 tens and 7 ones.

Stage 4: Long division

The layout below, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.

Conventionally the 20, or 2 tens, and the 8 ones forming the answer are recorded above the line.

How many packs of 15 can we make from 432 biscuits?

432 ÷ 15 becomes

$$\begin{array}{rrrrr} & & 2 & 8 & \text{r } 12 \\ 1\ 5 & 4 & 3 & 2 & \\ & 3 & 0 & 0 & \\ \hline & 1 & 3 & 2 & \\ & 1 & 2 & 0 & \\ \hline & & 1 & 2 & \end{array}$$

432 ÷ 15 becomes

$$\begin{array}{rrrrl} & & 2 & 8 & \\ 1\ 5 & 4 & 3 & 2 & \\ & 3 & 0 & 0 & 15\times20 \\ \hline & 1 & 3 & 2 & \\ & 1 & 2 & 0 & 15\times8 \\ \hline & & 1 & 2 & \end{array}$$

432 ÷ 15 becomes

$$\begin{array}{rrrrr} & 2 & 8 & \cdot & 8 \\ 1\ 5 & 4 & 3 & 2 & \cdot\ 0 \\ & 3 & 0 & \downarrow & \downarrow \\ \hline & 1 & 3 & 2 & \\ & 1 & 2 & 0 & \\ \hline & & 1 & 2 & 0 \\ & & 1 & 2 & 0 \\ \hline & & & & 0 \end{array}$$

Remainders can be expressed in three ways:

1. As a remainder r 12 Answer 28 r 12
2. As a fraction 12/15 = 4/5 Answer 28 4/5
3. As a decimal by continuing the division process Answer 28.8 (to a given number of decimal places)
